ACOUSTIC BEHAVIOUR OF ELLIPTICAL MUFFLERS WITH SINGLE-INLET AND DOUBLE-OUTLET

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1. Introduction

- Elliptical muffler with single-inlet and double-outlet
- Reduces flow noise and back pressure
- Similarities and differences in comparison with circular geometries. Dependence on the eccentricity.
- Three-dimensional acoustic modelling: Mode-matching method
  FEM
1. Introduction

- Acoustic behaviour

- Analytical models
  - Plane wave:
    - Important limitations
- Numerical models
  - Three-dimensional:
    - Simple geometries
- Measurements
  - Experimental set-up:
    - Expensive
    - Validation
2. Mathematical approach: MMM

- **Mode-matching method**

- **Solutions of the wave equation**
  - **Circular ducts**
    \[ P_A(r_1, \phi_1, z_1) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( A_{m,n}^+ e^{-jk_{A,m,n}z_1} + A_{m,n}^- e^{jk_{A,m,n}z_1} \right) J_m \left( \alpha_{m,n} \frac{r_1}{R_1} \right) \cos(m\phi_1) \]
  - **Elliptical chamber**
    \[ P_B(v, u, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( B_{m,n}^+ e^{-jk_{B,m,n}z} + B_{m,n}^- e^{jk_{B,m,n}z} \right) c_\alpha e_m(v, q_{m,n}) C_\alpha e_m(u, q_{m,n}) \]

θᵢ = 90, 180 degrees

Acoustic Behaviour of Elliptical Mufflers with Single-Inlet and Double-Outlet
2. Mathematical approach: MMM

- First step. Pressure and velocity conditions at discontinuities
  Expansion
  \[ P_A\big|_{z_1=0} = P_B\big|_{z=0}, \quad U_A\big|_{z_1=0} = U_B\big|_{z=0} \quad \text{on} \quad S_A \]
  \[ U_B\big|_{z=0} = 0 \quad \text{on} \quad S_B - S_A \]
  Contraction
  \[ P_B\big|_{z=L} = P_C\big|_{z_2=0}, \quad U_B\big|_{z=L} = U_C\big|_{z_2=0} \quad \text{on} \quad S_C \]
  \[ P_B\big|_{z=L} = P_D\big|_{z_3=0}, \quad U_B\big|_{z=L} = U_D\big|_{z_3=0} \quad \text{on} \quad S_D \]
  \[ U_B\big|_{z=L} = 0 \quad \text{on} \quad S_B - S_C - S_D \]

- Second step. Multiplication by weighting functions and integration over cross-section

Pressure conditions: modes of circular ducts
  \[ J_t\left(\alpha_{t,s} \frac{r_i}{R_i}\right)\cos(t\varphi_i) \]

Velocity conditions: modes of elliptical chamber
  \[ ce_t(v, q_{t,s}) Ce_t(u, q_{t,s}) \]
2. Mathematical approach: MMM

- Second step. Details

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Pressure conditions: $3 (t + 1) (s + 1)$ equations
Velocity conditions: $2 (t + 1) (s + 1)$ equations
2. Mathematical approach: MMM

- Third step. Evaluation of integrals

Pressure. Circular ducts

- Orthogonality of Fourier-Bessel functions

- Analytical integrals

\[ \int_0^R J_s(\lambda r) J_s(\mu r) r \, dr = \begin{cases} \frac{R}{\lambda^2 - \mu^2} (\mu J_s(\lambda R) J'_s(\mu r) - \lambda J_s(\mu R) J'_s(\lambda r)) & \lambda \neq \mu \\ \frac{R^2}{2} \left( J'_s(\lambda r)^2 + \left(1 - \frac{s^2}{\lambda^2 R^2}\right) J_s(\lambda r)^2 \right) & \lambda = \mu \end{cases} \]

Velocity. Elliptical chamber

- Orthogonality of Mathieu functions

- Analytical integrals

\[ \int_0^{2\pi} e_{2s}(v, q)^2 \cos(2v) \, dv = \pi \left( A_{2s}^{(2s)} + \sum_{r=0}^{\infty} A_{2r+1}^{(2s)} \right) \]

\[ \int_0^{2\pi} e_{2s+1}(v, q)^2 \cos(2v) \, dv = \frac{\pi}{2} \left( A_{2s+1}^{(2s+1)} \right)^2 + \pi \sum_{r=0}^{\infty} A_{2r+1}^{(2s+1)} A_{2r+3}^{(2s+3)} \]

\[ \int_0^{2\pi} e_s(v, q)^2 \, dv = \pi \]

- Numerical integrals

\[ \int_0^{\pi} C_s(u, q)^2 \cosh(2u) \, du ; \quad \int_0^{\pi} C_s(u, q)^2 \, du \]
2. Mathematical approach: MMM

- Third step. Evaluation of integrals

  Pressure. Elliptical chamber
  
  - Mathieu functions in terms of Fourier-Bessel functions
  
  $$ce_{2x}(v,q)Ce_{2x}(u,q) = \frac{ce_{2x}(0,q)ce_{2x}(\pi/2,q)}{A_0^{(2x)}} \sum_{t=0}^{\infty} (-1)^t A_{2t}^{(2x)} J_{2t}\left(\frac{2\sqrt{q}}{\rho} r\right) \cos(2t \theta)$$

  $$ce_{2x+1}(v,q)Ce_{2x+1}(u,q) = \frac{ce_{2x+1}(0,q)ce_{2x+1}(\pi/2,q)}{\sqrt{q} A_1^{(2x+1)}} \sum_{t=0}^{\infty} (-1)^t A_{2t+1}^{(2x+1)} J_{2t+1}\left(\frac{2\sqrt{q}}{\rho} r\right) \cos((2t+1) \theta)$$

  - Graf’s addition theorem
  
  $$J_m(\mu r \cos(m \theta)) = \sum_{j=-\infty}^{\infty} J_{m-j}(\mu r \cos(j \varphi_i + m \theta_i))$$

  Velocity. Circular ducts
  
  - Apply the same procedure

- Fourth step. Truncation of the algebraic system of equations

  Incident plane wave $$A_{0,0}^+ = 1$$ Anechoic terminations $$C_{m,n}^- = D_{m,n}^- = 0$$

  $$5 (t + 1) (s + 1)$$ equations $$5 (m + 1) (n + 1)$$ unknowns

  $$t = m = p$$ $$s = n = q$$
3. Experimental measurement

- Experimental set-up
  Transfer function method

- Acoustic attenuation performance
  Transmission loss

\[ TL = -10 \log \left( \frac{S_{33} H_r - H_{34}^2 S_C + S_{33} H_r - H_{34}^2 S_D}{S_{11} H_r - H_{12}^2 S_A} \right) \]

\[ H_r = e^{jk_0 s} \]

Acoustic Behaviour of Elliptical Mufflers with Single-Inlet and Double-Outlet
4. Results and discussion

- Validation: Transmission Loss vs frequency

$L = 0.3 \text{ m}, a = 0.23/2 \text{ m}, b = 0.13/2 \text{ m}, R_1 = R_2 = R_3 = 0.051/2 \text{ m}, \delta_1 = 0 \text{ m}, \delta_2 = \delta_3 = 0.073 \text{ m}, \theta_1 = \theta_2 = 0^\circ, \theta_3 = 180^\circ$
4. Results and discussion

Effect of length

$L = 0.05 \text{ m}, a = 0.23/2 \text{ m}, b = 0.13/2 \text{ m}, R_1 = R_2 = R_3 = 0.051/2 \text{ m}$

$L = 0.3 \text{ m}$ and $L = 0.15 \text{ m}, a = 0.23/2 \text{ m}, b = 0.13/2 \text{ m}, R_1 = R_2 = R_3 = 0.051/2 \text{ m}$
4. Results and discussion

- **Effect of eccentricity**

  \[ L = 0.25 \text{ m}, \ R_1 = R_2 = R_3 = 0.051/2 \text{ m}, \ \delta_1 = 0 \text{ m}, \ \delta_2 = \delta_3 = 0.06 \text{ m}, \ \theta_1 = \theta_2 = 0^\circ, \ \theta_3 = 180^\circ \]

  **First case:** \( a = 0.23/2 \text{ m}, \ b = 0.13/2 \text{ m}, \ \varepsilon_1 = 0.8249 \)

  **Second case:** \( a = 0.1858/2 \text{ m}, \ b = 0.1609/2 \text{ m}, \ \varepsilon_2 = 0.5 \)

  **Third case:** circular, \( R = 0.1729/2 \text{ m} \)

- **Cut-off frequencies**

  **Elliptical**

  \[ f_{c,2,0} = \frac{c_0}{2\pi} \sqrt{\frac{4q_{2,0}}{\rho^2}} \]

  \[ \rho = \varepsilon a \]

  1599 Hz \hspace{1cm} 1877 Hz

  **Circular**

  \[ f_{c,2,0} = \frac{3.05 c_0}{\pi 2 R} \]

  1909 Hz

Acoustic Behaviour of Elliptical Mufflers with Single-Inlet and Double-Outlet
4. Results and discussion

- **Location of ducts**

  \[ L = 0.3 \text{ m}, a = 0.23/2 \text{ m}, b = 0.13/2 \text{ m}, R_1 = R_2 = R_3 = 0.051/2 \text{ m} \]

  **First case**: \( \delta_1 = 0.05 \text{ m}, \delta_2 = 0.06 \text{ m}, \delta_3 = 0.03 \text{ m}, \theta_1 = \theta_2 = 0^\circ, \theta_3 = 180^\circ \)

  **Second case**: \( \delta_1 = 0 \text{ m}, \delta_2 = 0.06 \text{ m}, \delta_3 = 0.03 \text{ m}, \theta_1 = \theta_2 = 0^\circ, \theta_3 = 180^\circ \)

  **Third case**: \( \delta_1 = 0 \text{ m}, \delta_2 = 0.0479 \text{ m}, \delta_3 = 0.0479 \text{ m}, \theta_1 = \theta_2 = 0^\circ, \theta_3 = 180^\circ \)

- **Comparison double-outlet/single-outlet**

Acoustic Behaviour of Elliptical Mufflers with Single-Inlet and Double-Outlet
5. Conclusions

- The mode-matching method has been applied to the acoustic analysis of elliptical mufflers with single-inlet and double-outlet.
- Comparisons with FEM and experimental measurements show good agreement.
- The effect of the chamber length, the eccentricity and the location of the ducts has been illustrated.
- The use of a second outlet pipe modifies the behaviour of short chambers, from transversal resonance to dome-like behaviour.
- The location of the outlet ducts on the nodal line of the mode $m = 2$ and $n = 0$ improves the acoustic attenuation.
- For similar configurations, the attenuation is slightly reduced in comparison with the single-outlet chamber.