Selection of Best Pre-Control Technique by Optimization Tools

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Abstract

In this work, our aim is to design an automatic system to select the parameters which determine the performance of some quality control tools. Particularly, we address the case of pre-control. Our scheme is applied in two different contexts. First, when a user intends to use pre-control as a test for the deviation of the process. Next, we present the selection model when the user is going to use pre-control to test the potential capability of the process. We propose the use of optimization techniques of MINLP, and we discuss several proposals and the corresponding results.

Keywords: Quality Control, Process Capability, MINLP

1 Introduction and Scope

The technique of pre-control was designed to control the proportion of nonconforming units resulting from a process, in such a way that it is possible to detect when this proportion is becoming too large. That is, the main purpose of pre-control is to prevent defects in manufacture and to monitor the process in order to control that its capability remains large.

Pre-control was first introduced in 1954 by Satterthwaite [5], and in its original version is described by Shainin and Shainin [6]. Later, different versions of pre-control have been suggested in the literature (see Steiner [7]). Shainin and Shainin [6] remark that control charts are not adequate when the process has cycles or trends, and that a good alternative in these cases is pre-control. For a formal comparison of pre-control versus control charts, see Ledolter and Swersey [2]. Traver [8], discuss about the advantages of pre-control and Mackertich [3] shows that pre-control detects small deviations from the target more quickly than moving average control charts, for not very capable processes. A complete description of the pre-control technique can be found in Juran and Gryna [1].

Recently, several modifications over the start-up phase of pre-control have been presented in San Matías *et al.* [4]. These authors introduce the idea of making flexible the choice of certain parameters inherent to the PC qualification procedure, which are:

- The width of the green region, W_G
- The number k of consecutive green units needed to qualify the process
- The number t of consecutive yellow units needed to stop and adjust the process

In San Matías *et al.*, the following general PC qualification procedure is proposed: a width W_G for the green region is selected, and successive units of the process are measured until k consecutive greens are registered. If either a red or t consecutive yellows are obtained, the process is stopped and adjusted.

We may emphasize that:

- PC is a useful, simple to implement quality control tool
- the value of certain parameters highly determines the performance (efficiency) of the technique.
- the value of such parameters can be selected by the user

Having into account the strong influence of the choice of those parameters over the efficiency of PC, we think it is not advisable to let the whole decision to the user.

The goal of this work is to develop an effective and automated method to choose the PC parameters, so that some criteria may be optimized. That is, we propose the use of Optimization techniques in the context of Pre-control and Quality Control.

2 Preliminaries

In the following, we will consider a process whose quality characteristic has a target value μ_0, centered between the specifications limits LSL and USL. We will denote G_L and G_U the limits of the green area.

Consider the procedure proposed by San Matías *et al.* [4]. The general expression for the probability of qualifying the process *p(qualify)* after a certain number of inspections, given that a deviation δ occurs, is as follows:

$$p(qualify) \quad = \quad p1^k \cdot \frac{\sum_{h=0}^{t-1} p2^h}{1 - \sum_{h=1}^{t-1} p2^h \cdot \sum_{m=1}^{k-1} p1^m} \tag{1}$$

where

- $p1$ is the probability of registering a *green* unit
- $p2$ is the probability of registering a *yellow* unit

It is assumed that the process is normally distributed with a mean μ and standard deviation σ, being the potential capability index:

$$C_p = \frac{USL - LSL}{6\sigma}$$

Let us note that the width of the green region $W_G = G_U - G_L$. Instead of W_G, we will use a new parameter λ defined as:

$$\lambda = 2 \cdot \frac{USL - LSL}{W_G} \tag{2}$$

where $W_G = G_U - G_L$.

We have decided to find the values for the parameters that minimize the expected value of the sample size when the process is under control.

3 Pre-control as a test for deviations from the mean

The performance of pre-control is usually analyzed considering it as a test for the hypothesis set:

$$H_0 : \delta = 0 \quad \text{vs} \quad H_1 : \delta > 0$$

where δ is the standardized (unsigned) deviation of the process from the target value μ_0, that is:

$$\delta = \frac{|\mu_0 - \mu|}{\sigma}$$

If pre-control finishes by qualifying the process, we will accept that the deviation δ equals to 0. Otherwise, we will assume that it exists a certain non null deviation from the target. Then, for this test acceptance probability is the qualification probability given by (1), and then:

$$\alpha = 1 - p(qualify|\delta = 0)$$

It is possible to study the power of this test by examining the corresponding OC Curve, that in this case depends on the parameter δ, and we will use (1) to compute it.

This approach of pre-control implies that we are able to estimate the value of the potential capability index, $\hat{C}_p$. In the classical pre-control, it is usual to assume that the potential process capability is at the minimum acceptable value: usually $C_p = 1$, but sometimes $C_p = 1.33$ or even greater.

The probabilities $p1$ and $p2$ can be derived as a function of the potential capability, the process deviation from the target μ_0 and the width of the green region:

$$p1 = \Phi\left(\frac{6C_p}{\lambda} + \delta\right) - \Phi\left(-\frac{6C_p}{\lambda} + \delta\right) \quad (3)$$

$$p2 = \Phi\left(3C_p + \delta\right) - \Phi\left(-3C_p + \delta\right) - p1 \quad (4)$$

where

$$\delta = \frac{|\mu - \mu_0|}{\sigma}$$

Finally, using the equations (3) and (4) in (1), the general expression for the qualification probability is obtained.

For example, in Figure 1 we can see the OC Curves of pre-control when it is considered as a test for δ, using different values of k and t, with a fixed $\alpha = 0.05$. The process is supposed to have an estimated $\hat{C}_p = 1$.

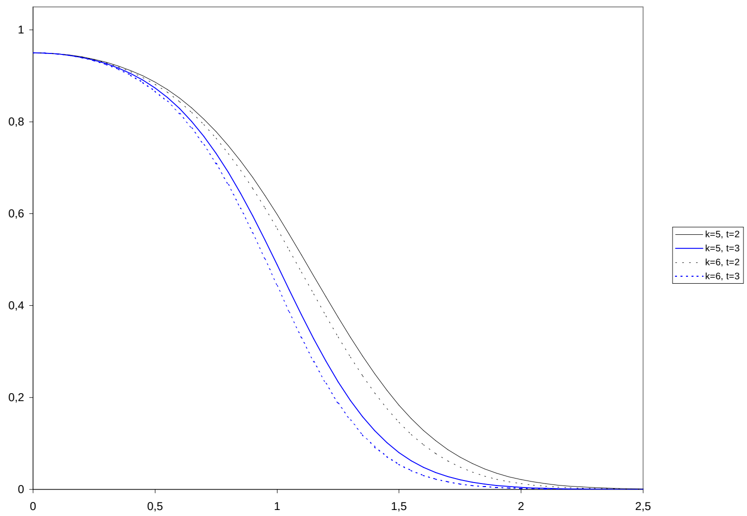

Figure 1: OC Curves of pre-control (as a test for δ) with $\alpha = 0.05$, in a process with $\hat{C}_p = 1$.

4 Pre-control as a test for the potential process capability

We can conceive the technique of pre-control with a different use, that is to consider it as a test for the potential capability of the process. Then, we deal with the hypothesis test:

$$H_0 : C_p \geq 1 \ \text{ vs } \ H_1 : C_p < 1$$

being C_p the potential capability index of the process. We will reject H_0 if the pre-control finishes with either a red unit or t yellows, and we will not reject the hypothesis if the pre-control qualifies the process.

Now, the OC Curve of the test is a function of C_p. Again, in order to obtain it we will use the same expression (1) for *p(qualify)* as in the previous section. However, in this case the expressions (3) and (4) for the probabilities $p1$ and $p2$ are adapted to work with a known estimation of δ':

$$\delta' = \frac{|\mu_0 - \mu|}{USL - LSL}$$

That is, we assume that it is possible to evaluate the proportion of absolute deviation from the target with respect to the specification interval. Note that:

$$\delta = 6C_p\,\delta'$$

Then:

$$p1 \;=\; \Phi\left(6C_p\left(\frac{1}{\lambda}+\delta'\right)\right)-\Phi\left(-6C_p\left(\frac{1}{\lambda}-\delta'\right)\right) \quad (5)$$

$$p2 \;=\; \Phi\left(3C_p\;(1+2\delta')\right)-\Phi\left(-3C_p\;(1-2\delta')\right)-p1 \quad (6)$$

The rate of false alarm for this test is:

$$\alpha \;=\; \sup_{C_p\geq 1}\left(1-p(qualify|C_p)\right) \;=\; 1-p(qualify|C_p=1)$$

In Figure 2, we show the OC Curves of pre-control when it is considered as a test for C_p, using different values of k and t, with a fixed $\alpha = 0.05$. The process is supposed to have an estimated $\delta' = 0$.

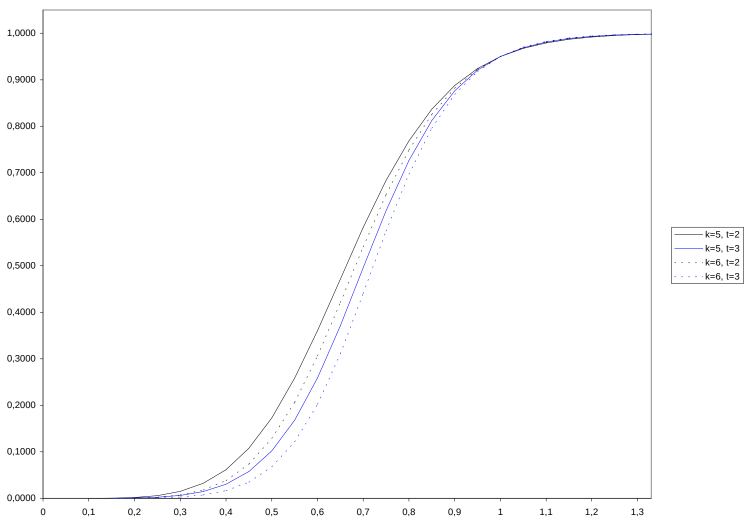

Figure 2: OC Curves of pre-control (as a test for C_p) with $\alpha = 0.05$, in a process with $\delta' = 0$.

5 Model

Let be n the number of units needed to finalize the qualification procedure. This number can be put in function of the problem parameters:

- k, the number of consecutive greens,
- t, the number of consecutive yellows
- $\lambda = 2 \cdot \frac{USL-LSL}{W_G}$, function of the width of the green area

A maximum value α^* for the rate of false alarm must be fixed. Also, it is necessary to establish a maximum error β^* for an intolerable deviation δ^*, such that in case the deviation takes that value, the probability $\beta(\delta^*)$ of qualifying the process be β^* at most.

Note that n_δ, α and $\beta(\delta^*)$ are functions of k, t and l.

Therefore, with objective expressed in the previous section, we model the problem as:

$$
\begin{aligned}
(P1) \qquad & \text{Min E(n)} & \\
& s.a. \;\; \alpha \leq \alpha^* & (7) \\
& \beta_{\delta^*} \leq \beta^* & (8) \\
& k, t \geq 0 \text{ and integer}, \lambda \geq 2 &
\end{aligned}
$$

Both the objective function and the restrictions are non linear functions of the parameters k, t and l, hence we are dealing with a MINLP.

5.1 Development of the objective function

Having into account that the probability function of the variable n can be written as:

$$
n = \begin{cases} 1 + n_{1V,F} & \text{with probability pv}_\delta \\ 1 + n_{1A,F} & \text{with probability pa}_\delta \\ 1 & \text{with probability pr}_\delta \end{cases}
$$

Then,

$$
E(n) = x_0 = \left(1 + \sum_{h=0}^{t-1} pa_\delta^h\right) \left(1 + \frac{\sum\limits_{i=1}^{k-1} pv_\delta^i \cdot \sum\limits_{j=0}^{t-1} pa_\delta^j}{1 - pa_\delta \cdot pv_\delta \cdot \sum\limits_{i=0}^{k-2} pv_\delta^i \cdot \sum\limits_{j=0}^{t-1} pa_\delta^j}\right)
$$

5.2 Constraints

The constraint (7) becomes:

$$1 - pv_0^k \cdot \frac{\sum_{h=0}^{t-1} pa_0^h}{1 - \sum_{h=1}^{t-1} pa_0^h \cdot \sum_{m=1}^{k-1} pv_0^m} \quad \leq \quad \alpha^*$$

With respect to (8), we have that:

$$pv_{\delta^*}^k \cdot \frac{\sum_{h=0}^{t-1} pa_{\delta^*}^h}{1 - \sum_{h=1}^{t-1} pa_{\delta^*}^h \cdot \sum_{m=1}^{k-1} pv_{\delta^*}^m} \quad \leq \quad \beta^*$$

6 Conclusions

In this work, we have presented a methodology to select the parameters which determine the performance of some quality control tools. Particularly, we address the case of pre-control. Our scheme is applied in two different contexts. First, when a user intends to use pre-control as a test for the deviation of the process. Next, we present the selection model when the user is going to use pre-control to test the potential capability of the process. In all cases, we work with the generalized version of Precontrol and optimization techniques of MINLP should be applied to solve the optimality problem.

7 Acknowledgements

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