Fuzzy inference system in a local eigenvector based colour image smoothing framework

Khleef Almutairi [1,3], Samuel Morillas [1], and Pedro Latorre-Carmona [2]

[1] Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València, Camino de Vera s/n, Valencia, 46022, Spain.
[2] Departamento de Ingeniería Informática, Universidad de Burgos, Avda. Cantabria s/n, Burgos, 09006, Spain.
[3] **Candidato a doctorado en Matemáticas, Álgebra, Geometría y Topología Grupo, UPV**

VIII Trobada d'Estudiants de Doctorat
VIII Encuentro de Estudiantes de Doctorado

1. Aim

Research in digital image processing and computer vision has remarkably increased in the last decade. In particular, image filtering has been a very active topic. Image Gaussian noise filtering methods are usually aimed at:

① Maximizing the smoothing of homogeneous regions.
② Gently smoothing of the edges and the details of the image.
③ Preserving the details and structure of the image.
④ Avoiding the introduction of colour artifacts in the processed image.

We hereby propose a method that uses a fuzzy inference system (FIS) [1] for colour image denoising. It is based on a previous transform from the RGB to an eigenvector-based space [2].

2. Image denoising with a Fuzzy Inference System

The process could be summarised in the following steps:

❶ Assume F is an RGB colour image. For each pixel F_0, we extract $N \times N$ neighbourhood and build a D matrix.

❷ Obtain the eigenvalues using this expression $det(D^T D - \lambda I) = 0$, where T is the matrix transpose and I is the identity matrix.

❸ Compute the three associated eigenvectors using the eigenvalues expression $(D^T D - \lambda_i I)\, v_i = 0$, where $i = 1, 2, 3$ which are the colour channels.

❹ Obtain the matrix U by applying the transformation of the eigenvectors by the data matrix D.

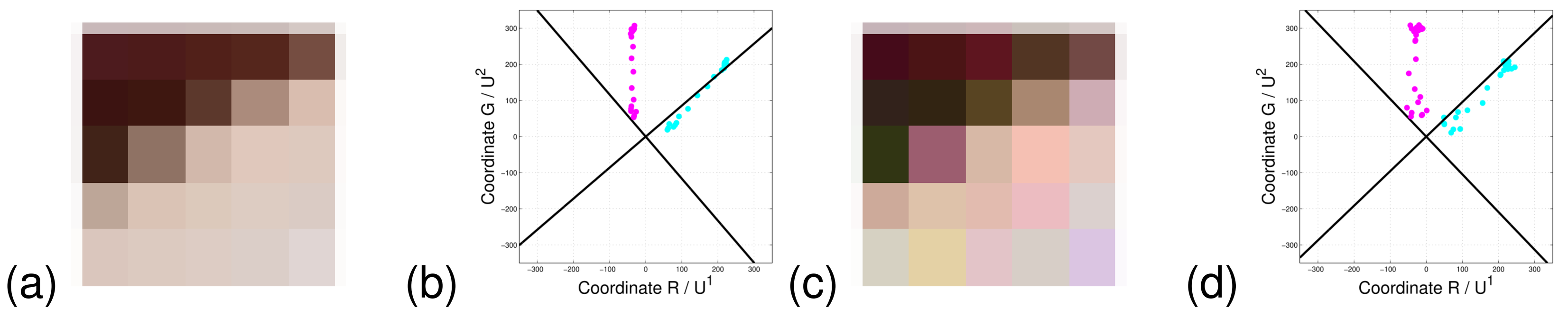

Fig. 1: (a) Image patch of a noise-free, and (c) noisy region in a part of the image with an edge. The corresponding RG coordinates (cyan dots) are shown in (b) and (d), along with the result of applying the eigenvector data transformation (magenta dots). The Black lines indicate the directions of the eigenvectors [2].

❺ Compute the three sample standard deviation values $\sigma(U^i)$ for each colour channel, and use them as crisp inputs for the FIS.

❻ Now we use a Fuzzy Logic Inference System to compute a series of smoothing coefficients, C_i from the values $\sigma(U^i)$ using the following implication rules for each i value, with $i = 1, 2, 3$:

① **IF** $\sigma(u^i)$ is low, **Then** the smoothing coefficient C_i is low.
② **IF** $\sigma(u^i)$ is medium, **Then** the smoothing coefficient C_i is medium.
③ **IF** $\sigma(u^i)$ is high, **Then** the smoothing coefficient C_i is high.

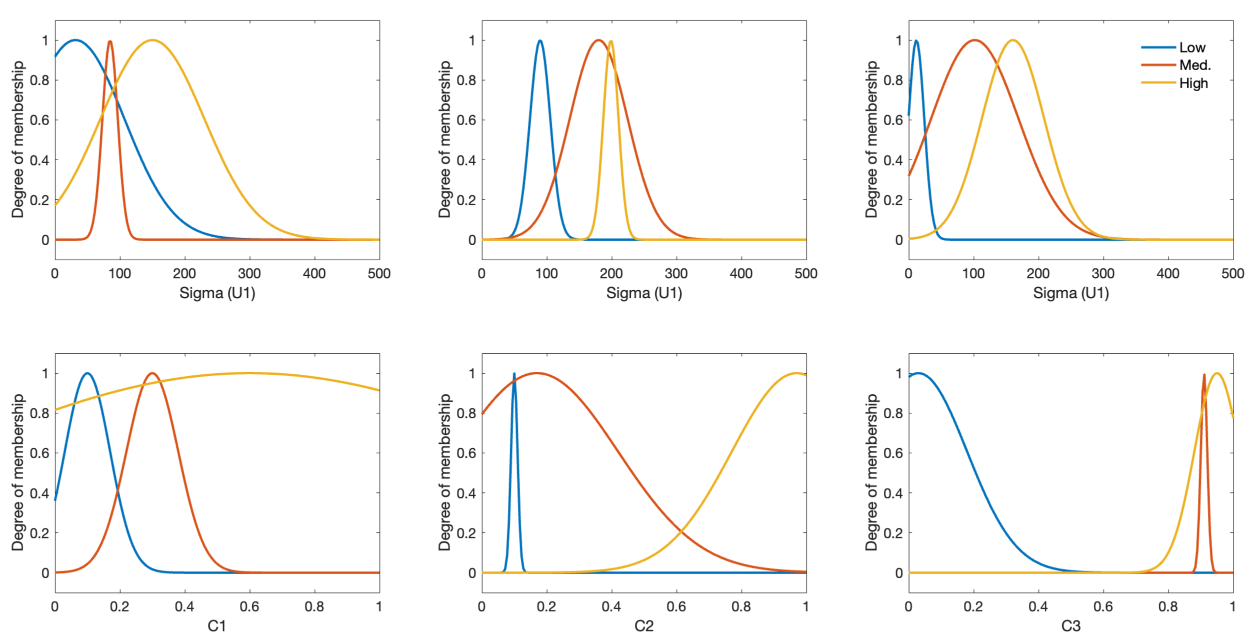

Fig. 2: Membership functions that are optimized to use for images with noise level 10 of the fuzzy inference system: The first row represents the input subsets, and the second row shows the output subsets.

❼ In order to smooth each component independently, $U_0 = (U_0^1, U_0^2, U_0^3)$ and W_p^i is needed to be set depending on the smoothing, we can compute that considering:

$$\hat{U}_0^i = \frac{\sum_{p=0}^{N^2-1} W_p^i U_p^i}{\sum_{p=0}^{N^2-1} W_p^i}, \quad W_p^i = exp\left(-\frac{|U_p^i - U_0^i|\; C_i}{D}\right), \quad i = 1, 2, 3,$$

Where p is the pixel numbers around the pixel intended to smooth, and D is the filter parameter.

❽ Converting the F_0 to the RGB colour space: $\hat{F}_0 = \hat{U}_0 V^T$.

3. Experimental results

The experimental steps of the proposed method are:

❶ Divide a set of images into training and validation sets.

❷ Perform FIS parameters optimization using the training image set of the inputs and outputs of the MFs using Genetic algorithms (GA) [3].

❸ Assess the optimized FIS framework, using the validation set of images, with the following quality measures [4]: Mean absolute error (MAE), Peak signal-to-noise ratio (PSNR) and, Normalized colour difference (NCD).

❹ Compare the proposed method against state-of-art methods using two images, shown in Figure 3. Each filter was applied in a 3×3 filter window, and the parameter settings recommended by the corresponding authors were used for each approach. The comparison results are shown in Table 1.

Table 1: Performance for each one of the methods, in terms of MAE, PSNR, NCD $(\times 10^2)$ using Beach and Lenna images with different sizes, contaminated with various standard deviation s of Gaussian noise. In the table, the best result for each noise level and performance measure is highlighted in blue, and the second best is highlighted in red.

Filter	s = 10			s = 20			s = 30		
	MAE	PSNR	NCD	MAE	PSNR	NCD	MAE	PSNR	NCD
The results of Beach image 200×200									
None	7.6	28.4	15.2	14.9	22.5	29.2	21.8	19.2	41.6
GMS[3]	7.2	28.7	10.3	11.3	24.8	17.6	16.5	21.5	26.5
NGMS[3]	9.2	26.6	11.2	12.5	23.9	16.4	15.5	21.9	21.4
CWF	5.6	31.0	8.3	9.6	26.3	12.4	13.0	23.7	15.7
EIG	5.6	31.0	8.9	10.0	25.9	15.1	14.1	23.0	20.6
Proposed	5.7	30.9	8.9	10.0	26.0	14.3	13.8	23.2	19.6
The results of Lenna image 90×90									
None	7.6	28.3	9.1	14.9	22.5	17.5	21.9	19.2	25.8
GMS[3]	4.9	31.6	5.0	8.5	27.0	9.2	13.7	22.9	15.3
NGMS[3]	5.1	31.2	4.9	7.7	27.6	7.7	10.5	25.1	11.0
CWF	3.4	35.2	3.0	5.1	31.5	4.3	6.9	28.9	5.9
EIG	4.6	32.6	4.7	8.2	27.6	8.5	11.4	24.6	12.1
Proposed	4.3	33.1	4.3	7.3	28.6	7.4	10.9	25.0	11.4

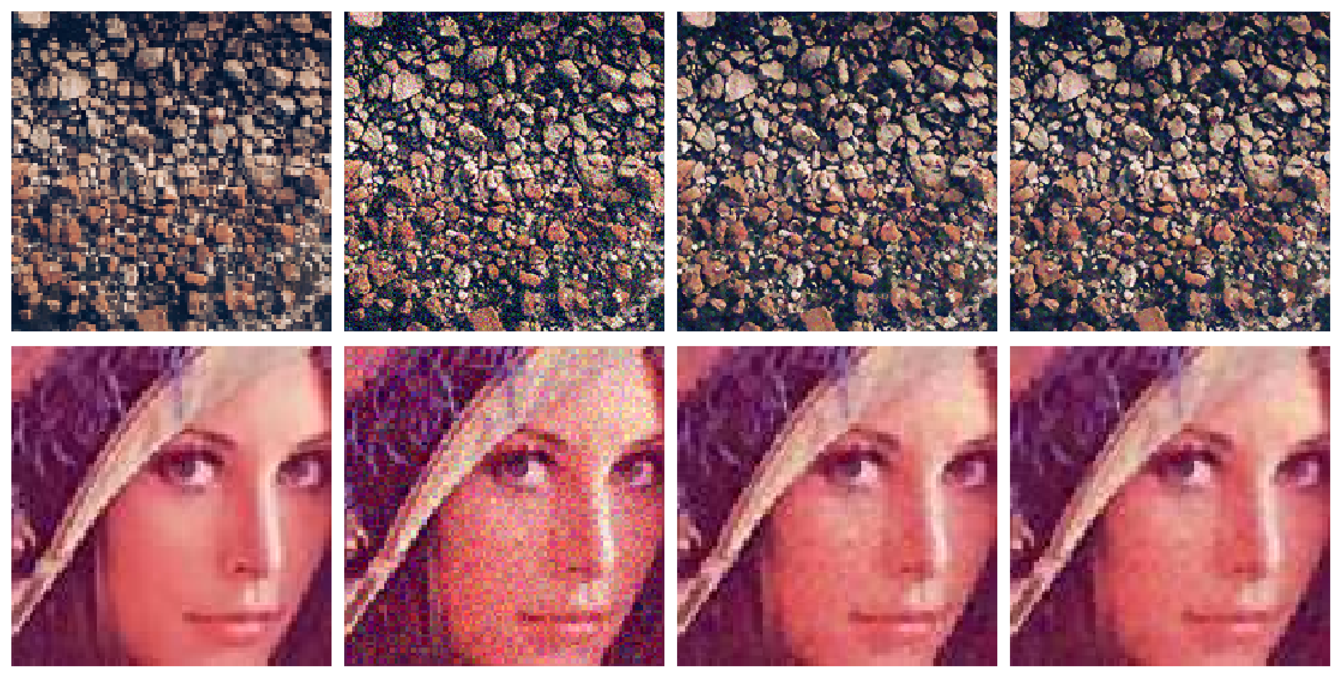

Fig. 3: Filtering methods output for visual comparison of Beach image in size 200×200 with noise level s = 30 in the first row. The second row is the Lenna image in size 90×90 with noise level $s = 20$. The first column on the left is the original images, the second column is the noisy images, the third column is the output images of the EIG method, and the last column is the output images of the proposed method.

4. Conclusions and future work

✧ This study uses a FIS framework, on top of an eigenvector-based image denoising strategy to filter Gaussian noise in colour images.

✧ The system infers the amount of noise that should be smoothed on a pixel-by-pixel basis. It performs competitively in terms of both noise reduction, as well as image structure and details preservation.

✧ The proposed method is better than the EIG filter in larger images with more homogeneous areas. However, CWF has the best results for high-spatial frequency content areas and images.

✧ This filtering method could be improved by training the system from data extracted from the images themselves.

✧ Analysis of parameter tuning valid for all levels of noise, simultaneously, is in order.

References

[1] Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.

[2] Pedro Latorre-Carmona, Juan-José Miñana, and Samuel Morillas. Colour image denoising by eigenvector analysis of neighbourhood colour samples. *Signal, Image and Video Processing*, 14(3):483–490, 2020.

[3] P Moallem, BS Mousavi, and S Sh Naghibzadeh. Fuzzy inference system optimized by genetic algorithm for robust face and pose detection. *Int. J. Artif. Intell*, 13(2):73–88, 2015.

[4] Konstantinos N Plataniotis and Anastasios N Venetsanopoulos. Companion image processing software. In *Color Image Processing and Applications*, pages 349–352. Springer, 2000.

⁎ **Acknowledgments**: *S. Morillas acknowledges the support of the research project PID2019-107790RB-C22, funded by MCIN/ AEI/10.13039/501100011033/*