SIS-type epidemiological models.

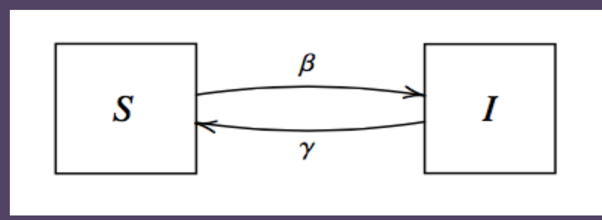
Ana Navarro Quiles.
Programa Doctorado en Matemáticas.

M.-C. Casabán[2]; J.-C. Cortés[1]; J.-V. Romero[2]; M.-D. Roselló[2]; R.-J. Villanueva.

1. Director. 2. Colaboradores.
Instituto Universitario de Matemática Multidisciplinar
Universitat Politècnica de València

Motivation

SIS-type epidemiological models constitute mathematical representations to describe the spread by individual-to-individual contact of infectious diseases. SIS models are useful for modelling diseases whose infection does not confer immunity, that is, any susceptible (S) who has been infected (I) can recover from the disease and then each individual becomes susceptible (S) again.



Examples of diseases that have been modelled by SIS models include gonorrhea, meningitis… and SIS models have also been used to model the dynamics of unhealthy lifestyle habits such as the excess of weight. In spite of their mathematical simplicity, SIS models constitute the basis of more refined and sophisticated models where, for instance, mode of transmission, resistanve, enviromental and cultutal factors, and further diseases characteristics can be considered

Formulation

SIS models are formulates as initial value problems based on nonlinear systems of differential equations of the form

$$\left\{ \begin{array}{lcl} S'(t) & = & -\beta S(t) I(t) + \gamma I(t), \\ I'(t) & = & \beta S(t) I(t) - \gamma I(t), \end{array} \right. \quad t > 0,$$

with initial conditions

$$S(0) = S_0, \quad I(0) = I_0,$$

where, S(t) and I(t) denote the percentage of susceptibles and infected at the time instant t, respectively. At the initial time instants, t=0, these values correspond to S_0 and I_0, respectively. The parameters β>0 and γ>0 denote the rate of decline in the percentage of susceptibles and the rate of infected that recover form the disease, respectively.

Objetive and theorical result

The principal objective is to obtain the one probability density function (1-PDF) of the solution of the SIS model, when we consider all the inputs as a random variables. With this aim we will use the Random Variable Transformation (RVT) method. This technique will also be applied to calculate the PDF of the time until a given proportion of the population remains susceptile/infected and an important epidemiological quantity called the basic reproductive number, usually denoted by R_0.

Theorically the 1-PDF of the number of susceptibles is given by

$$f_1(s,t) = \int_{\mathcal{D}_\gamma} \int_{\mathcal{D}_\beta} f_{S_0,\gamma,\beta}\left(\frac{\xi + e^{(\xi-\eta)t}(-1+s)\xi - s\eta}{\xi + e^{(\xi-\eta)t}(-1+s)\eta - s\eta}, \xi, \eta \right) \frac{e^{(\xi-\eta)t}(\xi-\eta)^2}{(\xi + e^{(\xi-\eta)t}(-1+s)\eta - s\eta)^2} \, \mathrm{d}\eta \, \mathrm{d}\xi \, .$$

Practical example

We apply SIS model to study the evolution of smoking among the Spanish men aged over 16 years old. In our context this population has been divided into two gorups, non-smokers (susceptible) and smokers (infected). We have the data corresponding with the period 1987-2006. We considers that the parameters have a particular distributions. Then, we obtain

- The 1-PDF of the solution S(t).
- The expectation and the standard deviation functions.
- Expectation and confidence intervals.
- The PDF of the time.

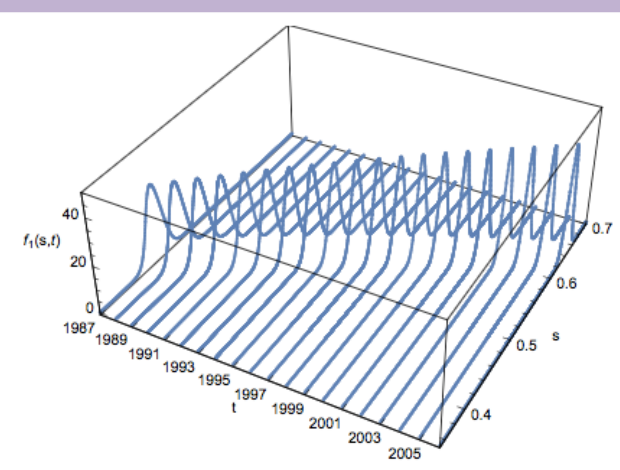


Figure 2: Plot of $f_1(s,t)$ in the Example 1 during the period 1987–2006 (corresponding to the solid lines).

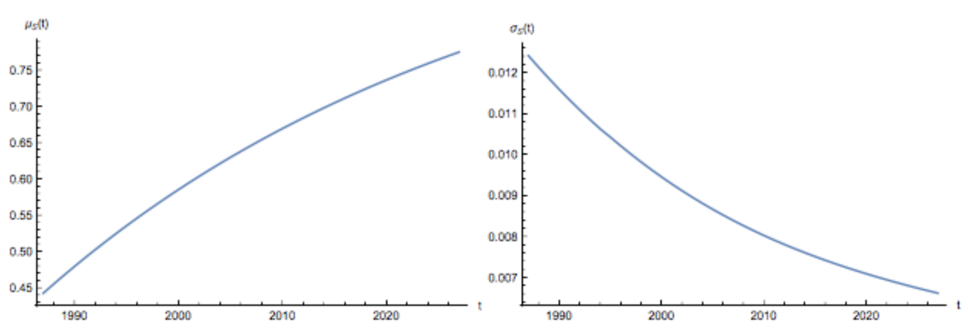


Figure 3: Plot of expectation function (left) and standard deviation function (right) in the Example 1.

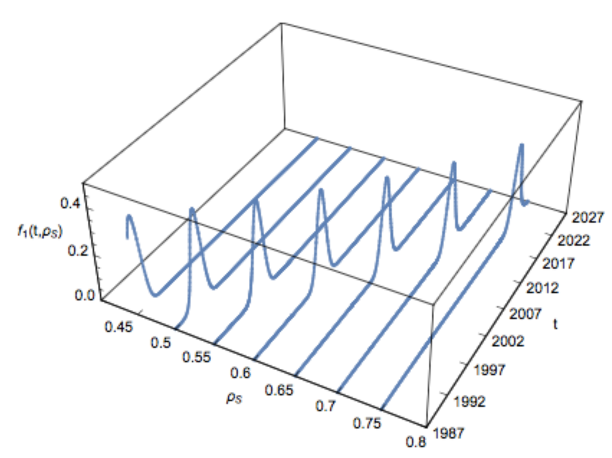


Figure 5: Plot of the 1-p.d.f. of the time T_S until a proportion $\rho_S \in \{0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75\}$ of the population remains susceptible in the Example 1.

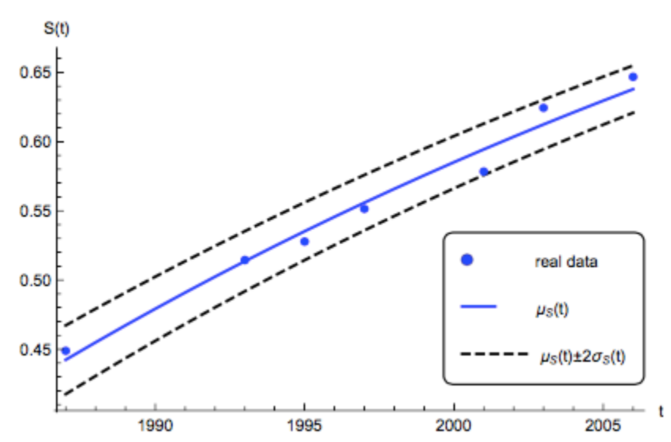


Figure 4: Expectation (solid line) and confidence intervals (dotted lines) in the Example 1.