A cluster method for Dynamic Life Tables

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1.INTRODUCTION

Mortality has decreased in all countries of Europe during the past century, presenting similar changing patterns. Despite these similar trends between countries, there are still considerable differences in mortality levels between eastern and western zone. The objective of this study is to present a method for detecting groups (clusters) of countries with similar mortality. This methodology is important in financial markets and especially for actuaries, to design annuities and life insurance. The method is based on Functional Data Analysis taking into account the geographical location of countries and, consequently, the neighborhood relationships among them. The method is applied to European countries.

2.1. Data

This study deals with mortality data of European countries for the period from 1990 to 2009. The age range considered is between 0 to 110+. Data has been taken from the Human Mortality Database for a total of 26 countries: Austria, Belarus, Belgium, Bulgaria, Czech republic, Denmark, Estonia, Finland, France, Germany, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Norway, Poland, Portugal, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom and Ukraine. Statistical analysis was performed using the R environment for statistical computing together with several R-packages.

2.2. Statistics

Standardized Mortality Ratio (SMR) is defined as the ratio between the observed number of deaths in a studied population and the expected number of deaths based on the age-specific rates in a standard population. If the SMR is greater than 1, there is "excess deaths" in the studied population.
The SMR is used to compare the mortality risk of a studied population and is defined as:

$$SMR_{i,j} = \frac{O_{i,j}(\text{Observed number of deaths})}{E_{i,j}(\text{Expected number of deaths})}$$

where i is the time and j the country. The Oi,j represents the number of deaths observed in each European country and Ei,j the deaths expected in each country if all countries had the same mortality as all Europe.

Global Moran's I is a summary measure of the intensity of the autocorrelation between the selected countries. Global Moran's I values vary between -1 and +1, but when the absolute value if the index is close or greater than 1, means the level of spatial autocorrelation is high.

$$I = \frac{N\sum_i\sum_j w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{(\sum_i\sum_j w_{ij})\sum_i (x_i-\bar{x})^2}$$

where N is the number of countries, $\bar{x}$ is the mean of the variable X, in our case the variable is SMR and W_{ij} is the matrix of spatial weights.

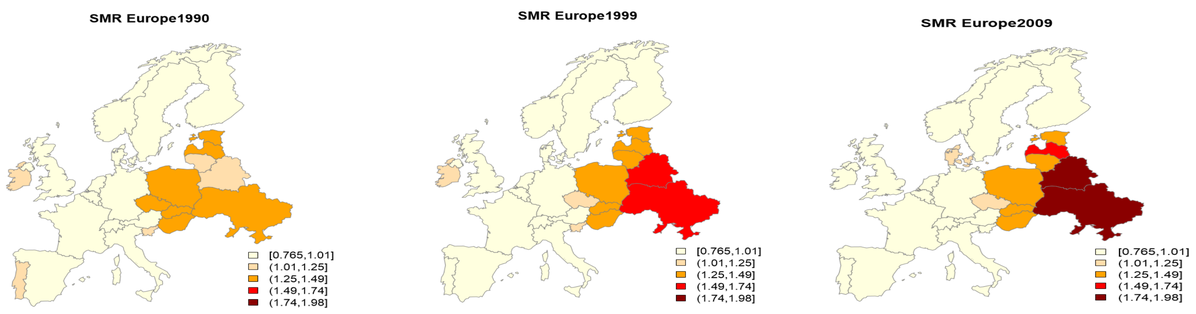
Local Moran's I gives information about the degree of correlation between selected countries, identifying the location of spatial clusters.

$$I = \frac{(x_i-\bar{x})}{\sum_i S_{x_i}^2}\cdot\sum_{j=1}^{N} w_{ij}(x_j-\bar{x})$$

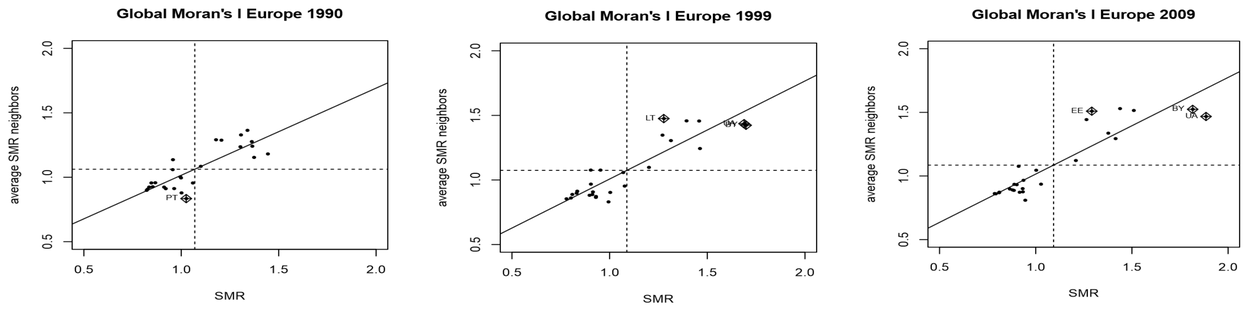
where $\sum S_{xi}^2 = (X_i-\bar{x})^2/N$. Local Moran's I values vary between -1 and +1 and it recognize four types of spatial clusters: I (H-H), II (L-H), III (L-L) and IV (H-L).
LISA Markov is related to both the directional LISA and the original space-time bivariate LISA. This relies on the quadrants of the Moran scatterplot which are now to define the states for a discrete Markov chain. The four quadrants are I (H-H), II (L-H), III (L-L), IV (H-L) with the first position indicating whether the observations are above or below the mean, while the second does the same but for the spatial lag. These four states give rise to 16 types of transitions.

2. RESULTS

The following graphs show the quintiles of the SMR for Europe. We can see that mortality is only increasing in Eastern countries over time while in other countries the mortality decreases as the color intensity is rising.



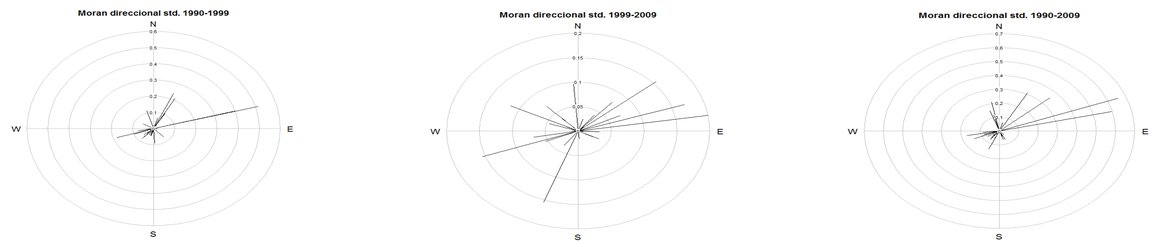
The graphics of Moran's I indicate that there is spatial positive correlation, because mostly of the observations are distributed in quadrants I and III.



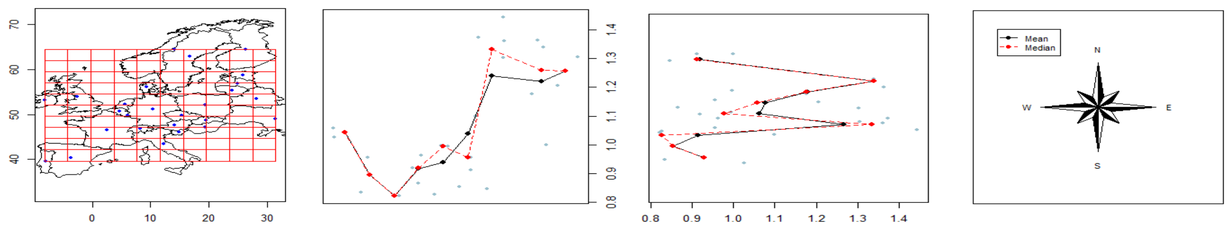
The next table shows the values of the Moran´s I. The p-values are significant indicating the existance of a spatial effect on mortality. (We only show 3 years, but all years are significant).

	Moran's I test under randomisation				Monte-Carlo simulation of Moran's I		
Año	I.Moran	Expectation	Variance	p-value	I.Moran	Nº simulations	p-value
1990	0,68	-0,04	0,03	0,00000997	0,68	1000	0,001
1999	0,76	-0,04	0,03	0,00000061	0,76	1000	0,001
2009	0,76	-0,04	0,03	0,00000040	0,76	1000	0,001

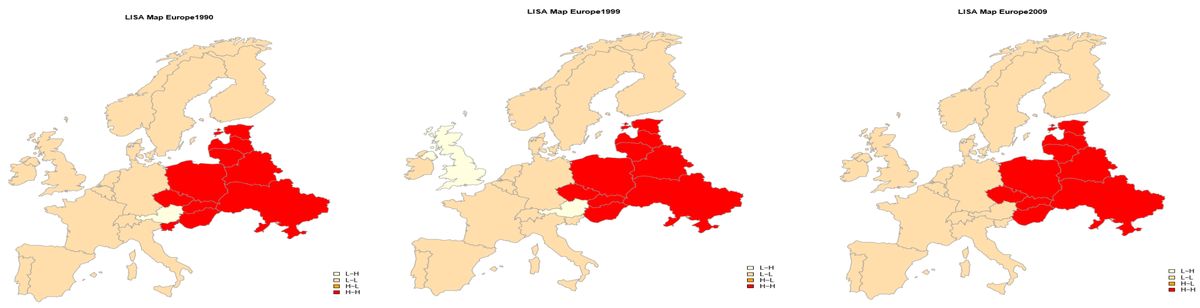
In the following figures the three directional Moran scatter plot for the comparison 1990-2009, 1990-1999 and 1999-2009 are shown. We can see that the mortality of Eastern Europe countries has increased over time as these countries are in quadrant I.
Another important thing to highlight in the last graph is that the number of countries from quadrant I is higher than in the other two. It means that the mortality is increasing in developed countries. The shift of these countries could be due to the actual financial crisis from 2007 to 2009.



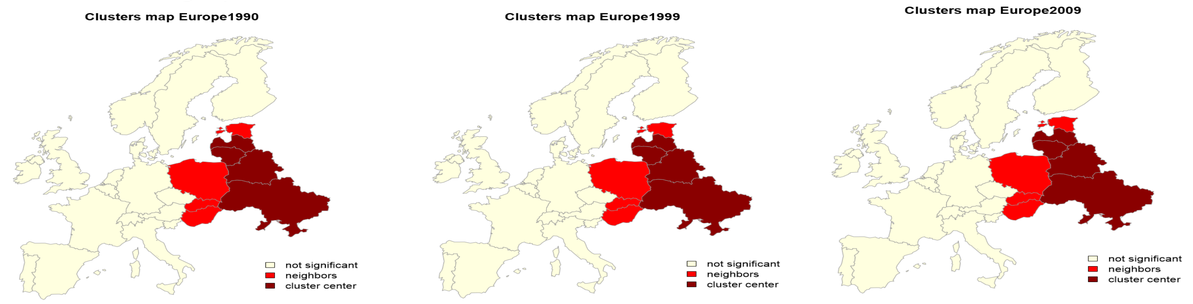
In the four drift maps we can see that the countries of Northwestern Europe have higher mortality than other countries.



In the three LISA maps we observe that two groups of countries, L-L and H-H predominate.



Finally, the significant cluster (H-H) consists of Ukraine, Belarus, Lithuania, Latvia, Estonia, Poland, Slovakia and Hungary .



The method of LISA Markov shows spatial-temporal dependence between the studied european countries, with a significant p-value (9.540167e-19).

Markov chain					Observed probabilities					Theoretical probabilities				
	LL	LH	HL	HH		LL	LH	HL	HH		LL	LH	HL	HH
LL	302	1	0	0	LL	0,9934	0,0033	0,0033	0,0000	LL	0,9904	0,0065	0,0031	0,0000
LH	2	10	0	0	LH	0,1667	0,8333	0,0000	0,0000	LH	0,0216	0,9753	0,0001	0,0031
HL	2	0	2	1	HL	0,4000	0,0000	0,4000	0,2000	HL	0,0112	0,0001	0,9824	0,0064
HH	0	0	2	171	HH	0,0000	0,0000	0,0116	0,9884	HH	0,0002	0,0110	0,0214	0,9674

In conclusion, the Global Moran's I and LISA Markov have shown that there is a spatial-temporal correlation in the mortality of European countries. Particularly, the countries of Eastern Europe have higher mortality than the rest. These countries form a significant cluster H-H.

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