UNIVERSITAT POLITÈCNICA DE VALÈNCIA

Departamento de Comunicaciones

Antenas

Tema 2
Fundamentos de radiación

Ecuaciones de Maxwell

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$$

Ecuaciones de onda para los campos

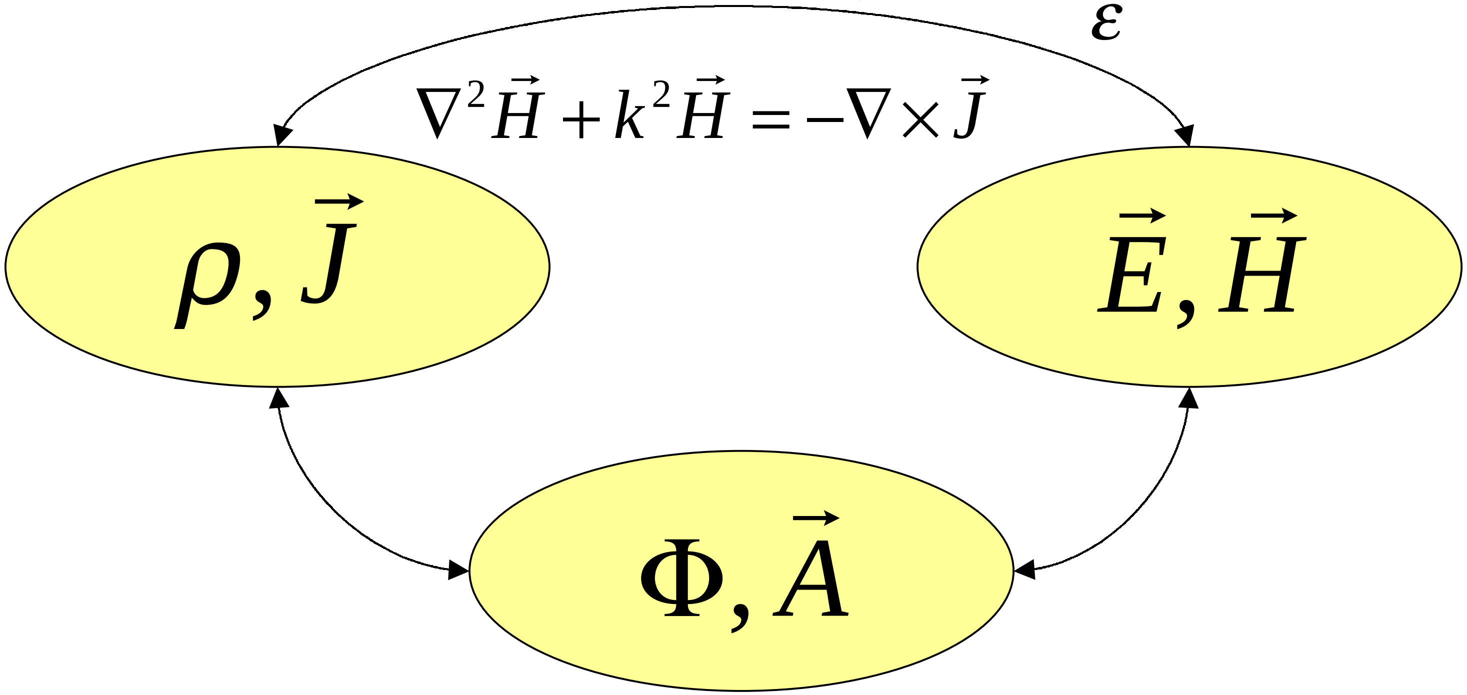
$$\nabla^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = j\omega\mu\vec{J} + \nabla\left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2\vec{H} + \omega^2\mu\varepsilon\vec{H} = -\nabla\times\vec{J}$$

Potenciales

$$\nabla^2 \vec{E} + k^2 \vec{E} = j\omega\mu \vec{J} + \nabla(\frac{\rho}{\varepsilon})$$

$$\nabla^2 \vec{H} + k^2 \vec{H} = -\nabla \times \vec{J}$$



Definición de los potenciales

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -j\omega\vec{A} - \nabla\Phi$$

Ecuaciones de onda para los potenciales

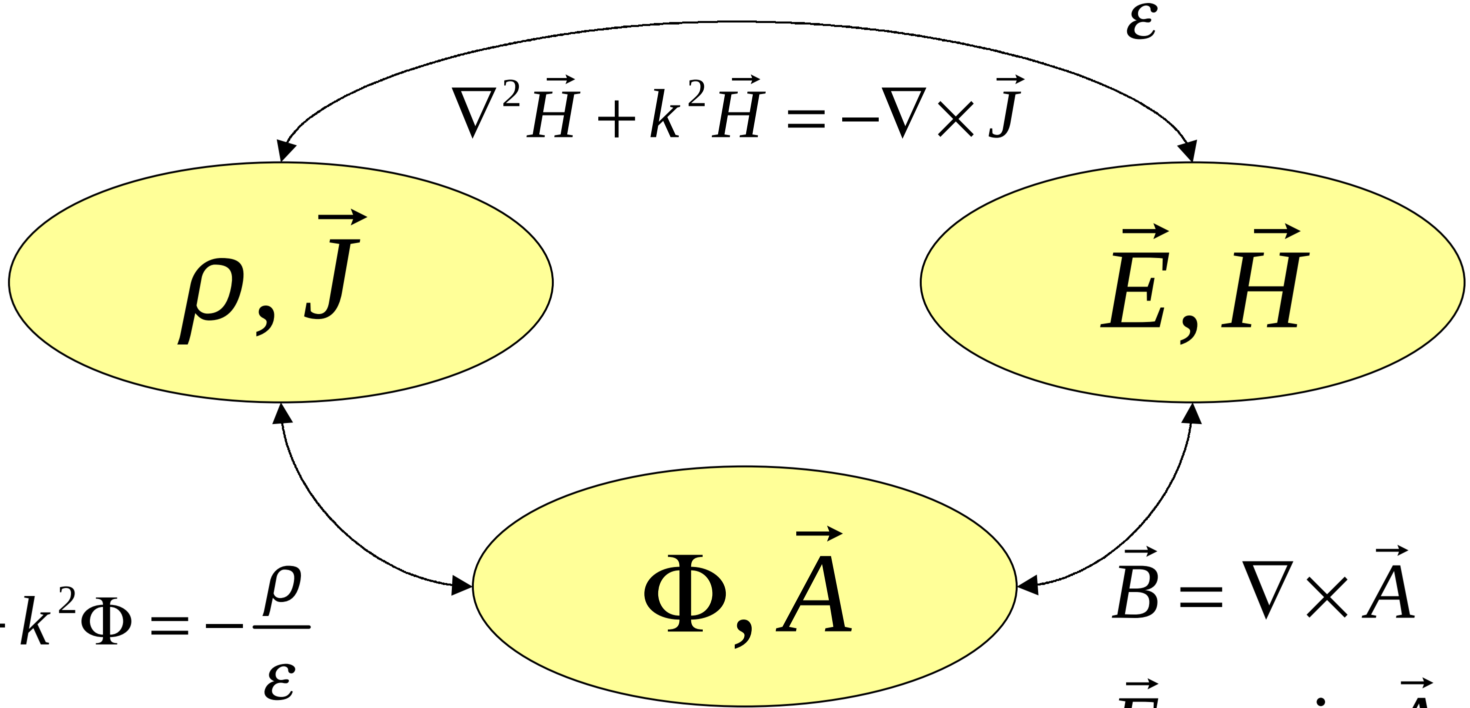
$$\nabla^2\Phi + k^2\Phi = -\frac{\rho}{\varepsilon}$$

$$\nabla^2\vec{A} + k^2\vec{A} = -\mu\vec{J}$$

Potenciales

$$\nabla^2\vec{E} + k^2\vec{E} = j\omega\mu\vec{J} + \nabla\left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2\vec{H} + k^2\vec{H} = -\nabla\times\vec{J}$$



$$\nabla^2\Phi + k^2\Phi = -\frac{\rho}{\varepsilon}$$

$$\nabla^2\vec{A} + k^2\vec{A} = -\mu\vec{J}$$

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -j\omega\vec{A} - \nabla\Phi$$

Soluciones de la ecuación de onda

$$\nabla^2\Omega + k^2\Omega = 0$$

$$\Omega = Ae^{-jkz} + Be^{jkz}$$

$$\Omega = AH_0^{(1)}(k\rho) + BH_0^{(2)}(k\rho)$$

$$\Omega = A\frac{e^{-jkr}}{4\pi r} + B\frac{e^{jkr}}{4\pi r}$$

Ecuación de onda inhomogénea

$$\nabla^2 G + k^2 G = -\partial(\vec{r} - \vec{r}\,')$$

$$G(r, r') = \frac{e^{-jk\left|\vec{r} - \vec{r}\,'\right|}}{4\pi\left|\vec{r} - \vec{r}\,'\right|}$$

Soluciones integrales para los potenciales

$$\nabla^2 \Phi + k^2 \Phi = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J}$$

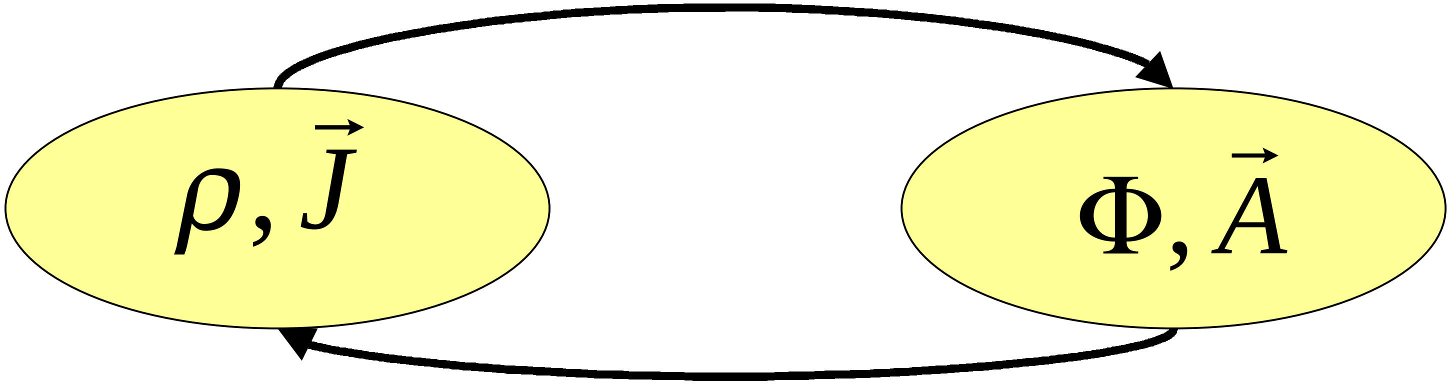
$$\Phi = \frac{1}{\varepsilon} \iiint_{v'} G(\vec{r}, \vec{r}\,') \rho(\vec{r}\,') dv'$$

$$\vec{A} = \mu \iiint_{v'} G(\vec{r}, \vec{r}\,') J(\vec{r}\,') dv'$$

Cálculo de los potenciales

$$\Phi = \frac{1}{\varepsilon} \iiint_{v'} G(\vec{r}, \vec{r}\,')\rho(\vec{r}\,')dv'$$

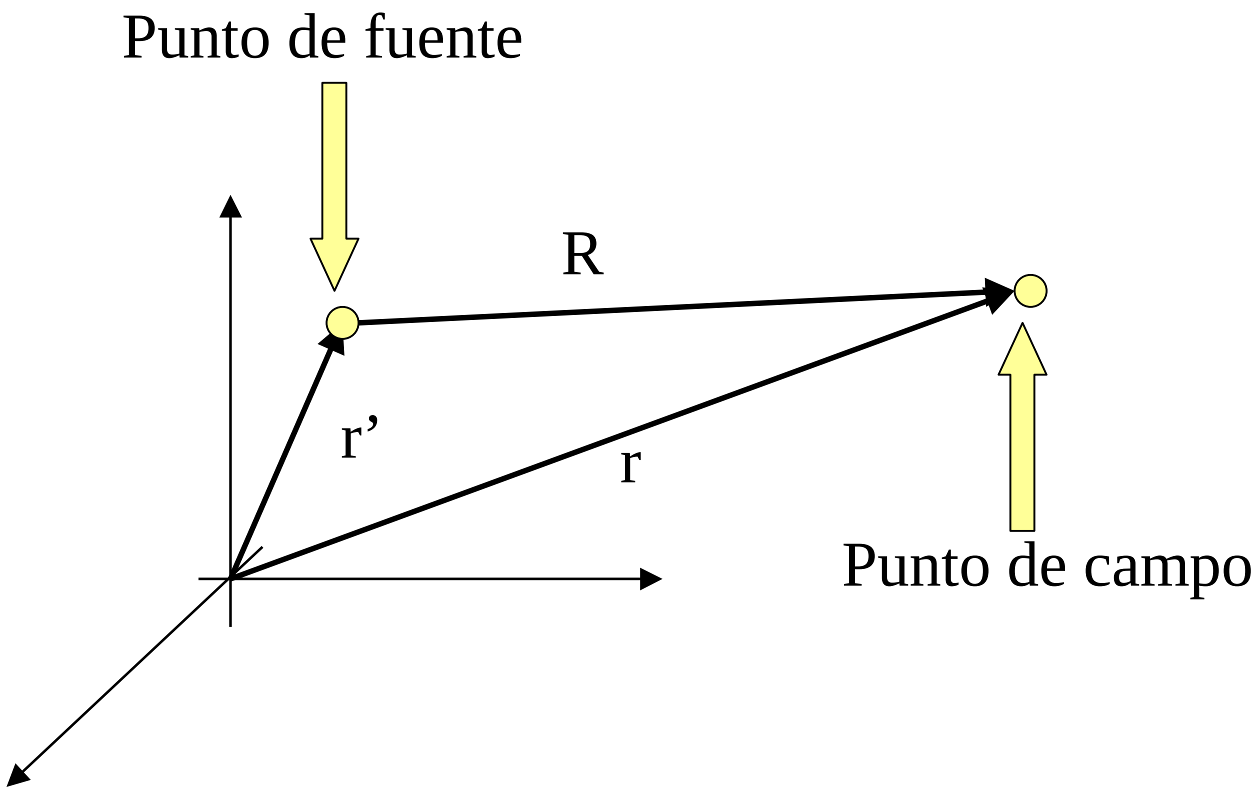
$$\vec{A} = \mu \iiint_{v'} G(\vec{r}, \vec{r}\,')J(\vec{r}\,')dv'$$



$$\nabla^2\Phi + k^2\Phi = -\frac{\rho}{\varepsilon}$$

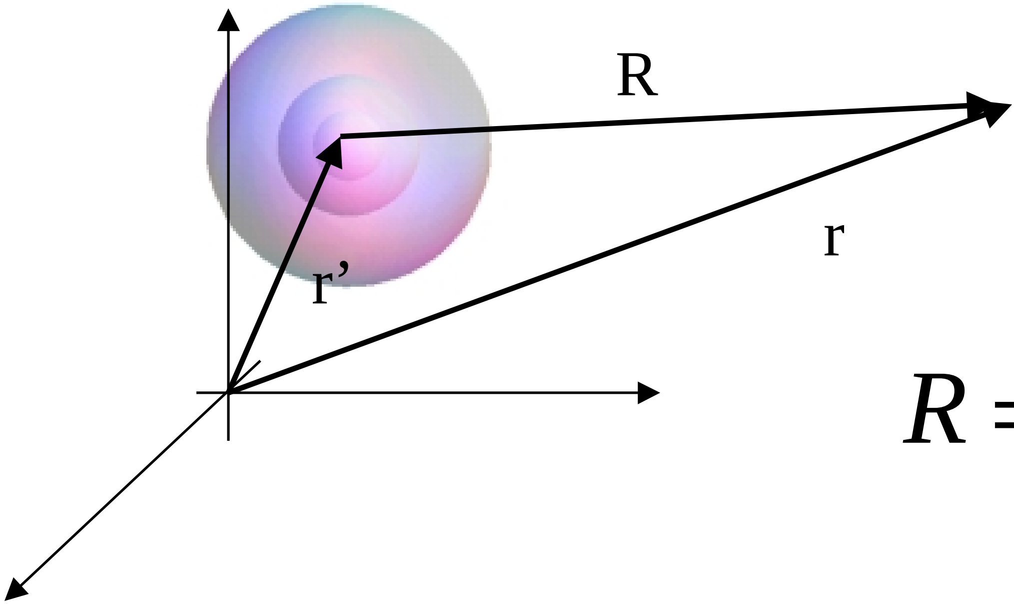
$$\nabla^2\vec{A} + k^2\vec{A} = -\mu\vec{J}$$

Nomenclatura para las Coordenadas de Campos y Fuentes



Función de Green

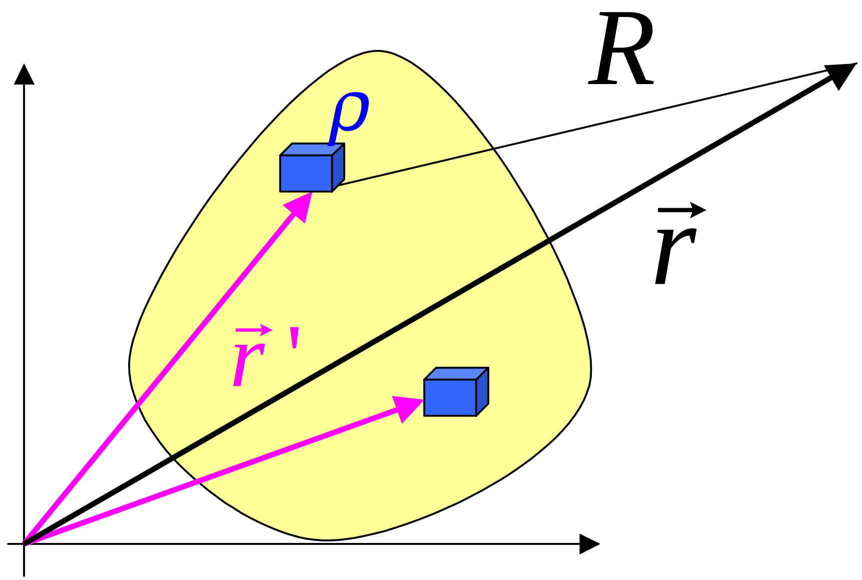
$$G(\vec{r},\vec{r}\,') = \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|} = \frac{e^{-jkR}}{4\pi R}$$



$$R = |\vec{r}-\vec{r}\,'|$$

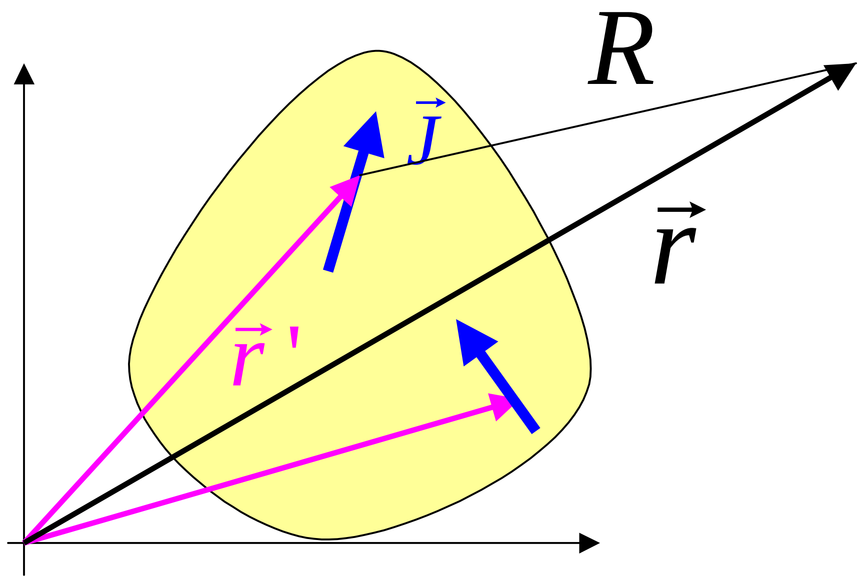
Potencial escalar

$$\Phi(\vec{r}) = \int_{v'} \rho(\vec{r}\,')G(\vec{r},\vec{r}\,')dv'$$



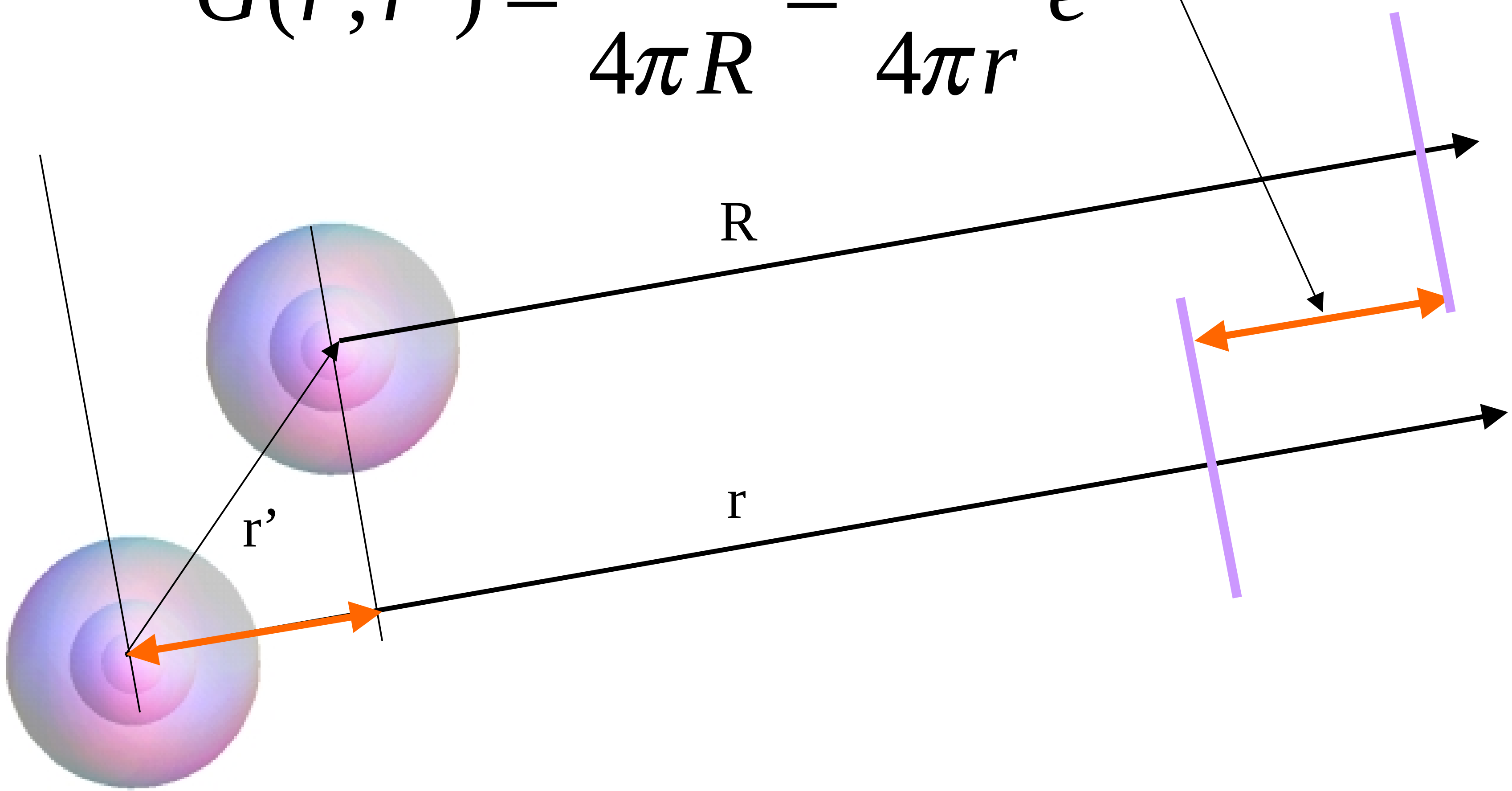
Potencial vector

$$\vec{A}(\vec{r}) = \int_{v'} \vec{J}(\vec{r}\,')G(\vec{r},\vec{r}\,')dv'$$



Función de Green aproximada

$$G(\vec{r},\vec{r}\,') = \frac{e^{-jkR}}{4\pi R} \simeq \frac{e^{-jkr}}{4\pi r} e^{jk\hat{r}\cdot\vec{r}\,'}$$



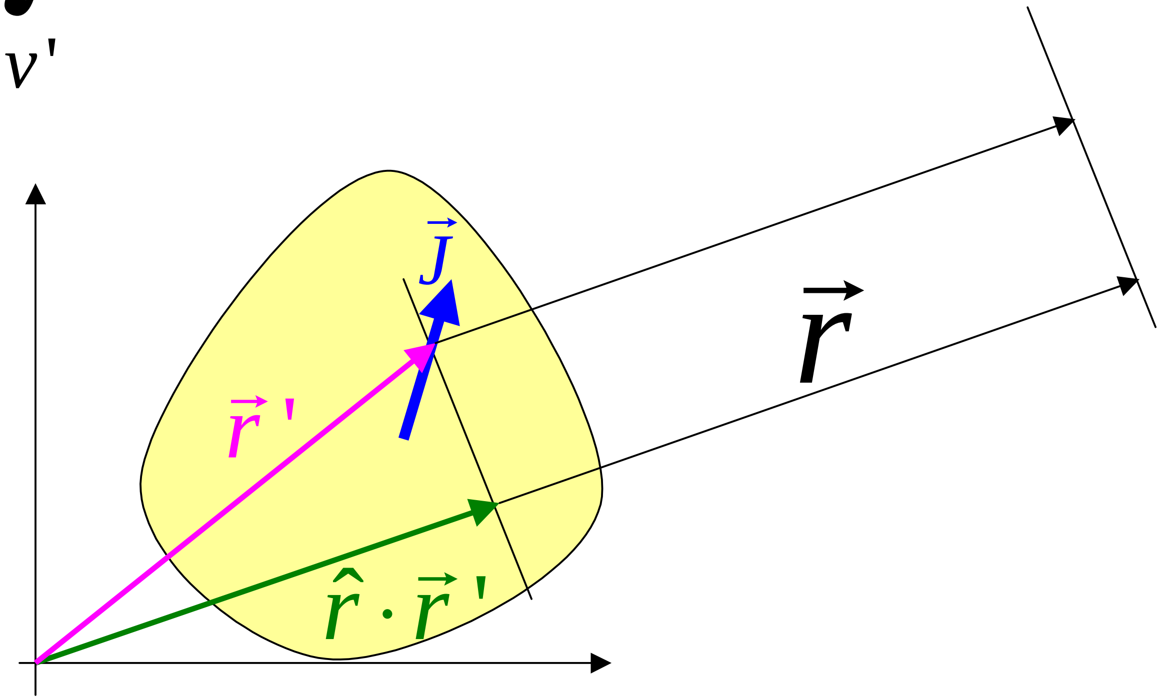
Potencial vector aproximado

$$\vec{A} = \frac{\mu e^{-jkr}}{4\pi r} \iiint_{v'} J(\vec{r}\,')e^{jk\vec{r}\cdot\hat{r}'} dv'$$

$$\vec{A} = \frac{\mu e^{-jkr}}{4\pi r} \vec{N}$$

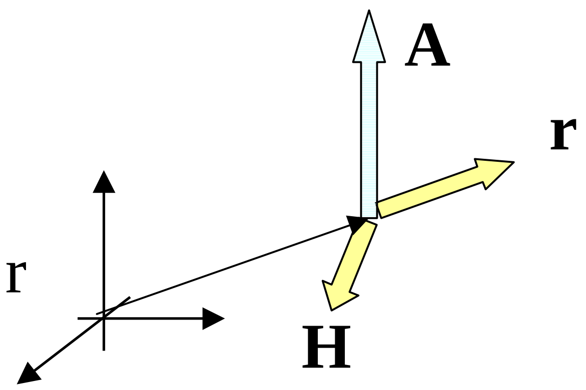
Vector de radiación

$$\vec{N} = \int_{v'} \vec{J}(\vec{r}\,')e^{jk\hat{r}\cdot\vec{r}\,'}dv'$$



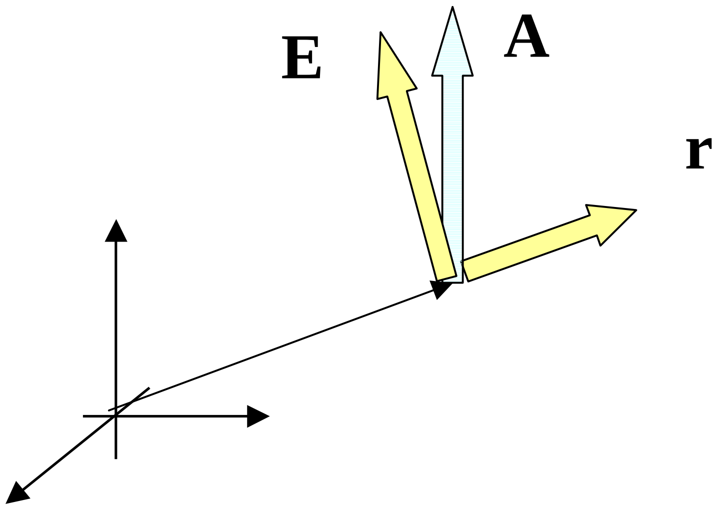
Campo magnético radiado

$$\vec{H} = -\frac{jk}{\mu}\hat{r}\times\vec{A}$$



Campo eléctrico radiado

$$\vec{E} = -j\omega(\vec{A} - \hat{r}\cdot\vec{A}) = j\omega(\hat{r}\times(r\times\vec{A}))$$



Campos radiados

$$\vec{E} = \eta(\vec{H} \times \hat{r})$$