

Operadores vectoriales

Coordenadas cartesianas

$$\nabla \psi = \hat{x} \frac{\partial \psi}{\partial x} + \hat{y} \frac{\partial \psi}{\partial y} + \hat{z} \frac{\partial \psi}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \hat{x} \nabla^2 A_x + \hat{y} \nabla^2 A_y + \hat{z} \nabla^2 A_z$$

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Coordenadas cilíndricas

$$\nabla \psi = \hat{\rho} \frac{\partial \psi}{\partial \rho} + \hat{\phi} \frac{\partial \psi}{\partial \phi} + \hat{z} \frac{\partial \psi}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho A_\rho + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\rho} \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \hat{z} \frac{1}{\rho} \left(\frac{\partial}{\partial \rho} \rho A_\phi - \frac{\partial A_\rho}{\partial \phi} \right)$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\rho} \frac{1}{\rho} & \hat{\phi} & \hat{z} \frac{1}{\rho} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla^2 \psi = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$$

$$\nabla^2 \mathbf{A} = \hat{\rho} \left(\frac{\partial^2 A_\rho}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial A_\rho}{\partial \rho} - \frac{A_\rho}{\rho^2} + \frac{1}{\rho^2} \frac{\partial^2 A_\rho}{\partial \phi^2} - \frac{2}{\rho^2} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial^2 A_\rho}{\partial z^2} \right)$$

$$+ \hat{\phi} \left(\frac{\partial^2 A_\phi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \rho} - \frac{A_\phi}{\rho^2} + \frac{1}{\rho^2} \frac{\partial^2 A_\phi}{\partial \phi^2} + \frac{2}{\rho^2} \frac{\partial A_\rho}{\partial \phi} + \frac{\partial^2 A_\phi}{\partial z^2} \right)$$

$$+ \hat{z} \left(\frac{\partial^2 A_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial A_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 A_z}{\partial \phi^2} + \frac{\partial^2 A_z}{\partial z^2} \right)$$

Operadores vectoriales

Coordenadas esféricas

$$\nabla \psi = \hat{r} \frac{\partial \psi}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial \psi}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \hat{r} \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right) + \hat{\theta} \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} r A_\phi \right) + \hat{\phi} \frac{1}{r} \left(\frac{\partial}{\partial r} r A_\theta - \frac{\partial A_r}{\partial \theta} \right)$$

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2}$$

$$\nabla^2 \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$$

$$\nabla^2 \mathbf{A} =$$

$$\hat{r} \left(\frac{\partial^2 A_r}{\partial r^2} + \frac{2}{r} \frac{\partial A_r}{\partial r} - \frac{2}{r^2} A_r + \frac{1}{r^2} \frac{\partial^2 A_r}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial A_r}{\partial \theta} + \frac{1}{r^2 \sin \theta} \frac{\partial^2 A_r}{\partial \phi^2} - \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} - \frac{2 \cot \theta}{r^2} A_\theta - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \right)$$

$$+ \hat{\theta} \left(\frac{\partial^2 A_\theta}{\partial r^2} + \frac{2}{r} \frac{\partial A_\theta}{\partial r} - \frac{A_\theta}{r^2 \sin^2 \theta} + \frac{1}{r^2} \frac{\partial^2 A_\theta}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial A_\theta}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 A_\theta}{\partial \phi^2} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \right)$$

$$+ \hat{\phi} \left(\frac{\partial^2 A_\phi}{\partial r^2} + \frac{2}{r} \frac{\partial A_\phi}{\partial r} - \frac{A_\phi}{r^2 \sin^2 \theta} + \frac{1}{r^2} \frac{\partial^2 A_\phi}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial A_\phi}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 A_\phi}{\partial \phi^2} + \frac{2}{r^2 \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial A_\theta}{\partial \phi} \right)$$